Poisson Probability Distribution

What is a **Poisson Probability Distribution**?

It is a probability distribution for a discrete random variable x, the number of successes in a fixed interval, with probability $P(x)$ such that the following conditions are met.

- The probability of two or more successes in any sufficiently smaller subinterval is 0.
- The probability of success for any two identical non overlapping interval is the same.
- The number of successes in any interval is independent from from other non overlapping interval with the same length.
- $0 \leq P(x) \leq 1$ and $\sum P(x) = 1$.

How to find the probability of a

Poisson Probability Distribution:

The **probability of x successes in an interval of fixed length with mean μ** is

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}, \text{ for } x = 0, 1, 2, 3, \cdots$$

with $e \approx 2.7183$ and $\sigma^2 = \mu$.

It is common to use the Greek letter lambda λ to represents the mean number of occurrences of the event in the given interval.

Example:

Consider a Poisson Probability Distribution for a discrete random variable x with mean $\mu = 10$ on a fixed interval.

- Find $P(x = 5)$.
- Find $P(x < 5)$.
- Find $P(x \geq 5)$.
- Find its variance σ^2.
- Find its standard deviation σ.

Solution:

- Find $P(x = 5) \Rightarrow P(x = 5) = \dfrac{10^5}{5!} e^{-10} = 0.038.$
- Find $P(x < 5) \Rightarrow P(x < 5) = P(x \leq 4).$
  $\Rightarrow P(x \leq 4) = P(x = 4) + P(x = 3) + \cdots + P(x = 0).$
  $\Rightarrow P(x < 5) = P(x \leq 4) = 0.029$
- Find $P(x \geq 5) \Rightarrow P(x \geq 5) = 1 - P(x \leq 4).$
  $\Rightarrow P(x \leq 4) = 0.029.$
  $\Rightarrow P(x \geq 5) = 1 - 0.029 = 0.971$
- Find its variance $\sigma^2 \Rightarrow \sigma^2 = \mu = 10.$
- Find its standard deviation $\sigma \Rightarrow \sigma = \sqrt{\sigma^2} = \sqrt{10} \approx 3.162.$

Poisson Probability Distributions & TI

When you have	Use TI command
$P(x = a)$	poissonpdf(μ, a)
$P(x \leq a)$	poissoncdf(μ, a)
$P(x \geq a)$	$1 -$ poissoncdf$(\mu, a - 1)$

You can find TI commands **poissonpdf** and **poissoncdf** by pressing [2ND], [VARS], then [↓] to locate them.

Example:

The phone calls to the college IT help desk occur at the rate of 1.5 per five minutes between 11:00am to 12:00 noon on Mondays. Compute the probability the number of these calls between 11:30am and 11:45am is

- exactly five.
- fewer than eight.
- at least four.

Solution:

This problem fits all criteria of a Poisson Probability Distribution with $\mu = 1.5 \cdot 3 = 4.5$ since the rate is given per five minutes, and our interval is 15 minutes.

Solution Continued:

Let x be the number of calls received by IT help desk during our desired interval.

Now we need to find

- exactly five. $\Rightarrow P(x = 5) = \text{poissonpdf}(4.5, 5) = 0.171$.
- fewer than eight.
  $\Rightarrow P(x < 8) = P(x \leq 7) = \text{poissoncdf}(4.5, 7) = 0.913$.
- at least four. $\Rightarrow P(x \geq 4) = 1 - P(x \leq 3)$
  $\Rightarrow 1 - \text{poissoncdf}(4.5, 3) = 0.658$.